Outline

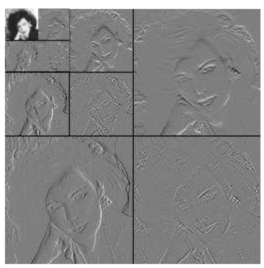
- Introduction
- Lossless compression
- Lossy compression
  - metrics
  - general methods
    - scalar
    - vector
    - differential
    - transform
      - Haar   Tutorial version
      - JPEG
      - wavelet
  - MPEG

Multiplication problem

XII * IX CVIII	transform → ← inverse transform	12 * 9 108	↓ useful application

Our useful application is QUANTIZATION and COMPRESSION!

General Methods - Video Subband Encoding HAAR Wavelets



HAAR Wavelets "averages and differences"

original 'image'   [8 4 1 3]   [a b c d]

transformed 'image' (level 1)   [6 2 2 -1] ($\underbrace{6\ 2}_{\text{averages}}$, $\underbrace{2\ -1}_{\text{details}}$) $\left[\frac{a+b}{2}\ \frac{c+d}{2}\ \frac{a-b}{2}\ \frac{c-d}{2}\right]$

transformed 'image' (level 2)   [4 2 2 -1] ($\underbrace{4}_{\text{average of averages}}$, $\underbrace{2}_{\text{detail}}$) $\left[\frac{a+b+c+d}{4}\ \frac{a+b-c-d}{4}\ \frac{a-b}{2}\ \frac{c-d}{2}\right]$

1-Dimension 2-level Haar Transform (N=4)

Step 1 – 1st level

1st level basis matrices N=4

[8 4 1 3] ● [(½ ½ 0 0) (0 0 ½ ½) (½ -½ 0 0) (0 0 ½ -½)]

= [6 2 2 -1]

Step 2 – 2nd level

2nd level basis matrices N=4

[6 2 2 -1] ● [(½ ½ 0 0) (½ -½ 0 0) (0 0 1 0) (0 0 0 1)]

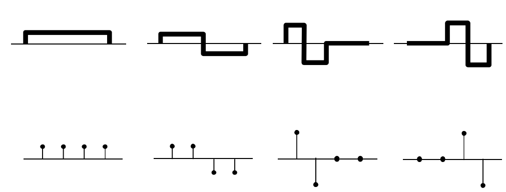
= [4 2 2 -1]

2-level Haar Basis Matrices N=4

[8 4 1 3] ●

[(¼ ¼ ¼ ¼) (¼ ¼ -¼ -¼) (½ -½ 0 0) (0 0 ½ -½)]

= [4 2 2 -1]



Deriving 2-level Haar Basis Matrices N=4

1st level basis matrices and 2nd level basis matrices

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2-level basis matrices

$$= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & 0 & -\frac{1}{2} \end{pmatrix}$$

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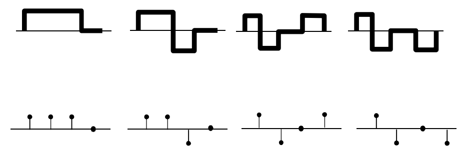
2-level Inverse Haar Basis Matrices N=4

$$[\,4\ \ 2\ \ 2\ \ {-1}\,] \bullet \begin{pmatrix} [1\ 1\ 1\ 0] & [1\ 1\ {-1}\ 0] & [1\ {-1}\ 0\ 1] & [1\ {-1}\ 0\ {-1}] \end{pmatrix}$$

$$= [\,8\ \ 4\ \ 1\ \ 3\,]$$



- Note: process is reversible without loss

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Forward transform • Inverse transform = ??

$$[\,8\ \ 4\ \ 1\ \ 3\,] \bullet \big(\text{transform}\big) \bullet \big(\text{transform}^{-1}\big) = [\,8\ \ 4\ \ 1\ \ 3\,]$$

$$\underbrace{\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & 0 & -\frac{1}{2} \end{pmatrix} \bullet \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}}_{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

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Compression - Quantization

[4 2 2 -1]

quantize — 25% compression ??

$$[\,4\ \ 2\ \ 2\ \ 0\,] \bullet \begin{pmatrix} [1\ 1\ 1\ 0] & [1\ 1\ {-1}\ 0] & [1\ {-1}\ 0\ 1] & [1\ {-1}\ 0\ {-1}] \end{pmatrix}$$

$$= [\,8\ \ 4\ \ 2\ \ 2\,]$$

What is SNR ?

original [8 4 1 3]
decoded [8 4 2 2]

$$\frac{(8^2 + 4^2 + 1^2 + 3^2)/4}{((8-8)^2 + (4-4)^2 + (1-2)^2 + (3-2)^2)/4} = \frac{90}{2} = 45$$

What would SNR be if we quantized to [4 2 0 -1] ?

Intuition: it's better to quantize values closer to 0.

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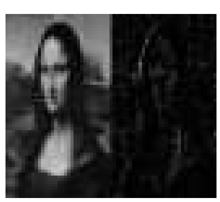
1-Dimension 1-level Haar Transform

before

after



200 200 200 202 …	200 201 … 0 -1 …
196 200 202 202 …	198 202 … -2 0 …
196 198 200 200 …	197 200 … -1 0 …
194 194 196 198 …	194 197 … 0 -1 …
…	…

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1-Dimension 3-level Haar Transform (N=8)

	12	8	10	6	-4	-2	8	-4
Step 1 – 1st level	10	8	-3	2	2	2	-1	6
Step 2 – 2nd level	9	-½	1	-2½	2	2	-1	6
Step 3 – 3rd level	4¼	4 ¾	1	-2½	2	2	-1	6

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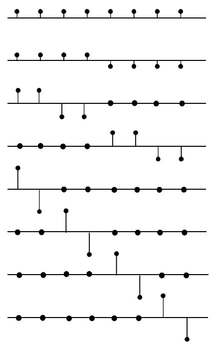
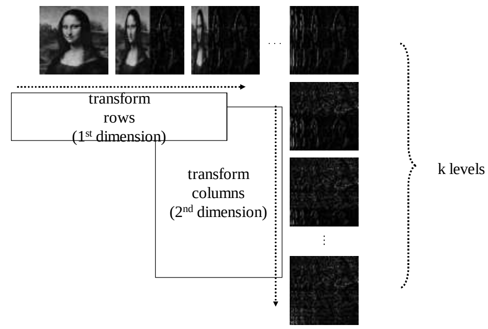
1-Dimension 3-level Haar Transform Basis Matrices (N=8)

row 0	1/8	(1 1 1 1 1 1 1 1)
row 1	1/8	(1 1 1 1 -1 -1 -1 -1)
row 2	1/4	(1 1 -1 -1 0 0 0 0)
row 3	1/4	(0 0 0 0 1 1 -1 -1)
row 4	1/2	(1 -1 0 0 0 0 0 0)
row 5	1/2	(0 0 1 -1 0 0 0 0)
row 6	1/2	(0 0 0 0 1 -1 0 0)
row 7	1/2	(0 0 0 0 0 0 1 -1)



2-Dimension Multilevel Haar Transform



Exercise: Perform a 2-D 2-level Haar standard transform on the following 4 X 4 pixel image

$$\begin{matrix}18&8&9&1\\20&6&11&15\\16&8&6&6\\6&6&-2&2\end{matrix} \Rightarrow \begin{matrix}13&5&5&4\\13&13&7&-2\\12&6&4&0\\6&0&0&-2\end{matrix} \Rightarrow \begin{matrix}9&4&5&4\\13&0&7&-2\\9&3&4&0\\3&3&0&-2\end{matrix}$$

$$\Downarrow$$

$$\begin{matrix}11&2&6&1\\6&3&2&-1\\-2&2&-1&3\\3&0&2&1\end{matrix}$$

$$\Downarrow$$

$$\begin{matrix}8.5&2.5&4&0\\2.5&-.5&2&1\\-2&2&-1&3\\3&0&2&1\end{matrix}$$

Exercise: 2-D 2-level Haar standard transform on 4 X 4 pixel image

$$\begin{matrix}18&8&9&1\\20&6&11&15\\16&8&6&6\\6&6&-2&2\end{matrix} \bullet \begin{pmatrix} (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14) & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14) & (\tfrac12\ -\tfrac12\ 0\ 0) & (0\ 0\ \tfrac12\ -\tfrac12) \\ (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14) & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14) & (\tfrac12\ -\tfrac12\ 0\ 0) & (0\ 0\ \tfrac12\ -\tfrac12) \\ (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14) & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14) & (\tfrac12\ -\tfrac12\ 0\ 0) & (0\ 0\ \tfrac12\ -\tfrac12) \\ (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14) & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14) & (\tfrac12\ -\tfrac12\ 0\ 0) & (0\ 0\ \tfrac12\ -\tfrac12) \end{pmatrix} \Rightarrow \begin{matrix}9&4&5&4\\13&0&7&-2\\9&3&4&0\\3&3&0&-2\end{matrix}$$

transform EACH row
with 1-D 2-level basis matrix

$$\bullet \begin{Bmatrix} (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14)^T & (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14)^T & (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14)^T & (\tfrac14\ \tfrac14\ \tfrac14\ \tfrac14)^T \\ (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14)^T & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14)^T & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14)^T & (\tfrac14\ \tfrac14\ -\tfrac14\ -\tfrac14)^T \\ (\tfrac12\ -\tfrac12\ 0\ 0)^T & (\tfrac12\ -\tfrac12\ 0\ 0)^T & (\tfrac12\ -\tfrac12\ 0\ 0)^T & (\tfrac12\ -\tfrac12\ 0\ 0)^T \\ (0\ 0\ \tfrac12\ -\tfrac12)^T & (0\ 0\ \tfrac12\ -\tfrac12)^T & (0\ 0\ \tfrac12\ -\tfrac12)^T & (0\ 0\ \tfrac12\ -\tfrac12)^T \end{Bmatrix}$$

transform EACH column
with rotated 1-D 2-level
basis matrix

$$\Downarrow$$

$$\begin{matrix}8.5&2.5&4&0\\2.5&-.5&2&1\\-2&2&-1&3\\3&0&2&1\end{matrix}$$

Exercise: Derive the 2-Dimension, 2-level Haar standard basis matrices for N=4

$$\begin{matrix}18&8&9&1\\20&6&11&15\\16&8&6&6\\6&6&-2&2\end{matrix} \bullet \left(\begin{matrix}
\tfrac{1}{16}\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} & \tfrac{1}{16}\begin{bmatrix}1&1&1&1\\1&1&1&1\\-1&-1&-1&-1\\-1&-1&-1&-1\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}1&1&1&1\\-1&-1&-1&-1\\0&0&0&0\\0&0&0&0\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&1&1&1\\-1&-1&-1&-1\end{bmatrix} \\
\tfrac{1}{16}\begin{bmatrix}1&1&-1&-1\\1&1&-1&-1\\1&1&-1&-1\\1&1&-1&-1\end{bmatrix} & \tfrac{1}{16}\begin{bmatrix}1&1&-1&-1\\1&1&-1&-1\\-1&-1&1&1\\-1&-1&1&1\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}1&1&-1&-1\\-1&-1&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&1&-1&-1\\-1&-1&1&1\end{bmatrix} \\
\tfrac{1}{8}\begin{bmatrix}1&-1&0&0\\1&-1&0&0\\1&-1&0&0\\1&-1&0&0\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}1&-1&0&0\\1&-1&0&0\\-1&1&0&0\\-1&1&0&0\end{bmatrix} & \tfrac{1}{4}\begin{bmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} & \tfrac{1}{4}\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&-1&0&0\\-1&1&0&0\end{bmatrix} \\
\tfrac{1}{8}\begin{bmatrix}0&0&1&-1\\0&0&1&-1\\0&0&1&-1\\0&0&1&-1\end{bmatrix} & \tfrac{1}{8}\begin{bmatrix}0&0&1&-1\\0&0&1&-1\\0&0&-1&1\\0&0&-1&1\end{bmatrix} & \tfrac{1}{4}\begin{bmatrix}0&0&1&-1\\0&0&-1&1\\0&0&0&0\\0&0&0&0\end{bmatrix} & \tfrac{1}{4}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&-1\\0&0&-1&1\end{bmatrix}
\end{matrix} \right)$$

$$= \begin{matrix}8.5&2.5&4&0\\2.5&-.5&2&1\\-2&2&-1&3\\3&0&2&1\end{matrix}$$

Compression

$$\begin{matrix}8.5&2.5&4&0\\2.5&-.5&2&1\\-2&2&-1&3\\3&0&2&1\end{matrix}$$

quantize ≤ |1.0|

$$\begin{matrix}8.5&2.5&4&0\\2.5&0&2&0\\-2&2&0&3\\3&0&2&0\end{matrix}$$

compression
6 out of 16

inverse $\Rightarrow$

reconstruction				original			
19.5	7.5	7.5	1.5	18	8	9	1
19.5	7.5	9.5	15.5	20	6	11	15
15.5	7.5	6.5	6.5	16	8	6	6
5.5	5.5	0.5	0.5	6	6	-2	2

What is SNR ?

$$\frac{(18^2 + 8^2 + 9^2 + \ldots + 2^2) / 16}{((18-19.5)^2 + (8-7.5)^2 + (9-7.5)^2 + \ldots + (2-0.5)^2) / 16}$$

$$= 6.703 = 8.3\text{dbs}$$

Exercise: Derive the 2-Dimension, 2-level Haar standard basis matrices for N=4

$$
\begin{matrix} 18 & 8 & 9 & 1 \\ 20 & 6 & 11 & 15 \\ 16 & 8 & 6 & 6 \\ 6 & 6 & -2 & 2 \end{matrix}
\bullet
\left(\begin{matrix}
\frac{1}{16}\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix} &
\frac{1}{16}\begin{pmatrix}1&1&1&1\\1&1&1&1\\-1&-1&-1&-1\\-1&-1&-1&-1\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}1&1&1&1\\-1&-1&-1&-1\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}0&0&0&0\\0&0&0&0\\1&1&1&1\\-1&-1&-1&-1\end{pmatrix} \\
\frac{1}{16}\begin{pmatrix}1&1&-1&-1\\1&1&-1&-1\\1&1&-1&-1\\1&1&-1&-1\end{pmatrix} &
\frac{1}{16}\begin{pmatrix}1&1&-1&-1\\1&1&-1&-1\\-1&-1&1&1\\-1&-1&1&1\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}1&1&-1&-1\\-1&-1&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}0&0&0&0\\0&0&0&0\\1&1&-1&-1\\-1&-1&1&1\end{pmatrix} \\
\frac{1}{8}\begin{pmatrix}1&-1&0&0\\1&-1&0&0\\1&-1&0&0\\1&-1&0&0\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}1&-1&0&0\\1&-1&0&0\\-1&1&0&0\\-1&1&0&0\end{pmatrix} &
\frac{1}{4}\begin{pmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{4}\begin{pmatrix}0&0&0&0\\0&0&0&0\\1&-1&0&0\\-1&1&0&0\end{pmatrix} \\
\frac{1}{8}\begin{pmatrix}0&0&1&-1\\0&0&1&-1\\0&0&1&-1\\0&0&1&-1\end{pmatrix} &
\frac{1}{8}\begin{pmatrix}0&0&1&-1\\0&0&1&-1\\0&0&-1&1\\0&0&-1&1\end{pmatrix} &
\frac{1}{4}\begin{pmatrix}0&0&1&-1\\0&0&-1&1\\0&0&0&0\\0&0&0&0\end{pmatrix} &
\frac{1}{4}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&-1\\0&0&-1&1\end{pmatrix}
\end{matrix}\right)
$$

Basis matrix (1,2)

$$
= \begin{matrix} 8.5 & 2.5 & 4 & 0 \\ 2.5 & -.5 & 2 & 1 \\ -2 & 2 & -1 & 3 \\ 3 & 0 & 2 & 1 \end{matrix}
$$

Exercise: Derive the 2-Dimension, 2-level Haar basis

Consider 2-Dimension 2-level basis matrix (1,2)

$$
\frac{1}{8}\begin{pmatrix}1&1&-1&-1\\-1&-1&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix} = \begin{pmatrix}1/8&1/8&-1/8&-1/8\\-1/8&-1/8&1/8&1/8\\0&0&0&0\\0&0&0&0\end{pmatrix}
$$

1-Dimension 2-level basis matrices

row 0 $\begin{bmatrix} ¼ & ¼ & ¼ & ¼ \end{bmatrix}$

row 1 $\begin{bmatrix} ¼ & ¼ & -¼ & -¼ \end{bmatrix}$

row 2 $\begin{bmatrix} ½ & -½ & 0 & 0 \end{bmatrix}$

row 3 $\begin{bmatrix} 0 & 0 & ½ & -½ \end{bmatrix}$

take <u>cross product</u> row 1 by <u>row</u> 2

X	¼	¼	-¼	-¼
½	1/8	1/8	-1/8	-1/8
-½	-1/8	-1/8	1/8	1/8
0	0	0	0	0
0	0	0	0	0

Haar Transform

original (256 X 256)

(a±b)/2

normalized

(a±b)/($2^{½}$)

compression 10 : 1

Normalized Haar Wavelet

using (a±b)/($2^{½}$)

original

25 : 1

100 : 1

Note: Images of Emmy Noether thanks to Prof Mulcahy (Spelman College)