CIS-280
Assignment 2: More Complex Algorithms 50 points
Due Tuesday, March 1, 2005

1. (25 points:) A function is unimodal over an interval (a, b) if it has only one hump — ie., the function has a maximum or a minimum, but not both. A trisection process, similar conceptually to the idea of interval halving, can be used to find the maximum of a function that is both unimodal on an interval (a, b) and has a maximum on that interval.

   One cannot determine the maximum by merely looking at the midpoint of the interval, because the relationship between f(a) and f(mid) does not say anything about whether the maximum is to the left or right of mid. However, if we trisect the interval with points x_1 and x_2, then where must the maximum lie if $f(x_1) < f(x_2)$? What if $f(x_1) > f(x_2)$? What if $f(x_1) = f(x_2)$? What part of the interval can be discarded in each of the cases?

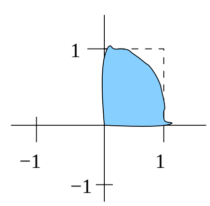
   Design a scheme function `Maxima` that takes as arguments the name of a function, the left endpoint of an interval, the right endpoint of an interval, and a value of ϵ, and returns the x-value at which the function is a maximum over the given interval (the answer that is returned should have an error of at most ϵ). Your function *Maxima* may call other functions that you design. You may assume that the function is unimodal over the given interval and has a maximum over that interval. Use your function to compute the following maximums:

$g(x) = -3x^2 + 2x - 4$	interval: (0, 1)	$\epsilon = .005$
$h(z) = -z^3 + 3.5z^2 + 6z + 3$	interval: (1, 8)	$\epsilon = .001$
$k(w) = -w^3/3 + 2.5w^2 + 14w - 2$	interval: (5, 15)	$\epsilon = .001$

2. (25 points:) The area of a circle is given by the formula $area = \pi r^2$
   Therefore, the area of a circle of radius 1 is

   $$area = \pi * 1^2 = \pi$$

   Suppose you do not remember the value of π. Then you can compute its value by calculating the area of a circle of radius 1, using a Monte Carlo method. In particular, one quadrant of a circle of radius 1 can be inscribed in a square whose sides are each length 1, as shown below.

   You can use a Monte Carlo technique to estimate the area of this quadrant of the circle, and then multiply by 4 to get an estimate of the area of a complete circle of radius 1. (Recall that the equation for a circle is $x^2 + y^2 = 1$. Thus a point (x,y) lies inside the circle if $x^2 + y^2 \leq 1$

   Design and implement a procedure `Est-Pi` that takes as argument the number of randomly generated points to be used in computing the area of the circle, and returns an estimate of the value of π. Thus your procedure will be defined as

   ```
   (define Est-Pi (lambda (n)
   ```

   Execute your procedure est-Pi for 1 trial, 10 trials, 50 trials, 100 trials, 500 trials. How close is your computed value to the actual value of π?

Copy your procedures into the submission web page, and submit them for grading for Homework-2. Be sure to click on `Confirm` so that it is sent to your TA.