

CIS-280
Assignment 3: Recursion versus Iteration
Due Tuesday, March 8, 2005

90 points:

(40 points for Scheme procedures, 50 points for complexity analysis)

Problems 1 and 2 should be submitted via the electronic submission system. The answers to Problems 3a-3d should be written on the assignment and turned in for grading to the instructor (not the TA).

1. (20 points) Design and implement two procedures `Dive1` and `Dive2` that take as arguments two positive integers `numa` and `numb` and return a count of the number of positions in which the digit in one of the numbers divides the digit in the other number exactly. For example, if the arguments are 1969483 and 9373893, both procedures return 5 since for the first, third, fourth, sixth, and seventh digit positions from the right, one of the digits divides the other exactly. (You may assume that neither `numa` nor `numb` contain the zero digit.) `Dive1` should be linear in time and space, and `Dive2` should be linear in time and constant in space.
2. (20 points) Design and test two procedures `CountDiv1` and `CountDiv2` that take as arguments three positive integers, `number`, `a`, and `b` (in that order, where $a \leq b$), and returns the sum of the integers between `a` and `b` inclusive that are exact divisors of `number`. For example,

(`CountDiv1 70 4 7`)

returns 12 since 5 and 7 are the only integers in the set $\{4, 5, 6, 7\}$ that divide 70 exactly. `CountDiv1` should be linear in time and space and `CountDiv2` should be linear in time but constant in space.

3. Complexity Analysis: 50 points

In the following problems, give the requested time or space complexity in Big O notation. (The primitive procedure random returns a randomly generated integer; you may assume that it has constant time complexity.) Be sure to specify what any parameters represent; that is, don't say it is $O(n)$ without saying what n represents. **Please write your answers on this assignment sheet and turn it in.**

We will not help you do these specific problems. However, we will review the answers in class or lab.

```
(a) (define addmult (lambda (x)
      ; x is a non-negative integer
      (cond ((= x 0) 0)
            (else (+ (compute (remainder x 10)) (addmult (quotient x 10)))))))

(define compute (lambda (y)
  (+ (* y y) (* y y y))))
```

What is the time complexity of procedure compute? _____

What is the space complexity of procedure compute? _____

What is the time complexity of procedure addmult? _____

```
(b) (define addalt (lambda (number)
      ; number is a non-negative integer
      (cond ((= number 0) 0)
            (else (+ number (addalt (quotient number 3)))))))
```

What is the time complexity of procedure addalt? _____

What is the space complexity of procedure addalt? _____

(Look on other side for Problems 3c and 3d.)

```
(c) (define Mine (lambda (a b)
      ; a is a non-negative integer
      ; b is a positive integer
      (cond ((= a 0) 1)
            (else (* (GetSum b) (Mine (- a 1) b))))))

(define GetSum (lambda (x)
      ; x is a positive integer
      (cond ((= x 1) 0)
            (else (+ (random 100) (GetSum (- x 1))))))
```

What is the time complexity of procedure GetSum? _____

What is the time complexity of procedure Mine? _____

```
(d) (define BigSum (lambda (number)
      ; number is a positive integer
      (SumIt (Expt3 number))))

(define SumIt (lambda(k)
      ; k is a positive integer
      (if (= k 0) 0 (+ k (SumIt (- k 1))))))

(define Expt3 (lambda (j)
      ; compute 3 to the j power
      (cond ((= j 0) 1)
            ((even? j) (square(Expt3 (/ j 2))))
            (else (* 3 (Expt3 (- j 1))))))

(define square (lambda (y)
      ; return the square of y
      (* y y)))
```

What is the time complexity of procedure Expt? _____

What is the time complexity of procedure SumIt? _____

What is the time complexity of procedure BigSum? _____