

NAME:

**CISC-304 First Midterm Exam: October 14, 2008**

**100 points**

**Please be careful to write clearly; answers that are not legible will not be graded.**

1. **SHORT ANSWER:**

- (a) (5 pts) If a formula A is satisfiable and a formula B is satisfiable, can the formula  $(A \wedge B)$  be unsatisfiable? If **YES**, give an example. If **NO**, explain why not.

- (b) (4 pts) Give two models for the following formula:

$$p \wedge (q \vee (p \rightarrow \neg r))$$

- (c) (4 pts) Suppose that a set of formulas  $S=\{A_1, A_2, A_3\}$  is unsatisfiable; what is the strongest statement that can be made about  $\neg(A_1 \wedge A_2 \wedge A_3)$ ?

- (d) (12 pts) (Circle TRUE or FALSE)

- i. A decision procedure for satisfiability is said to be sound if it always terminates with a decision. (TRUE or FALSE)
- ii. If A is a consequence of  $\neg B$  and A is unsatisfiable, then B must be a tautology. (TRUE or FALSE)
- iii. If assignment  $v$  is a model for A, and  $B \rightarrow A$  is a tautology, then  $v$  must be a model for B.

(e) (15 pts) Use three different decision procedures to determine whether the formula  $(p \vee q) \wedge \neg p$  is satisfiable. For each one, be sure to say how you know whether the formula is satisfiable using that decision procedure.

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2. (5 pts) Give a formula that is true if and only if either one or three of the atoms  $p_1, p_2, p_3, p_4$  are false. (Show your work and how you arrived at your formula.)

3. (10 pts) Use the method of semantic tableaux to determine whether the following formula is a tautology: (do not convert the formula to one without  $\rightarrow$  before drawing the semantic tableaux.)

$$(p_1 \wedge p_2) \rightarrow \neg(p_2 \rightarrow p_1)$$

Be sure to give your conclusion as to whether the formula is or is not a tautology and why.

4. (15 pts.) **Using the concept of assignments, formally prove** the following: (Do NOT use truth tables)  
If  $(A \rightarrow B) \wedge (C \vee D)$  is unsatisfiable and D is satisfiable, then B is not a tautology.

5. (15 pts) Using the substitution theorem and the importance equivalences that we discussed in class, prove the following equivalence: (You must show your work, including stating the names of the equivalences you are applying.)

$$(p \wedge \neg q) \vee \neg(p \vee q) \equiv \neg q$$

6. (15 pts) Prove via structural induction that every well-formed formula can be written using just the operators  $\neg$  and  $\rightarrow$ . (You **MUST** do the proof via structural induction — the goal of this problem is to determine whether you understand structural induction proofs.)