A1. Graphics (25 points) Shading

Given a triangle with vertices v_1, v_2, and v_3 and normal n_1, n_2, and n_3 respectively, assume the surface has material (k_a, k_d, k_s, shiny) for the ambient, diffuse, specular, and shinyness components, the eye is at the origin, and the light source is at p and has intensity (I_a, I_d, I_s) for the ambient, diffuse, and specular components.

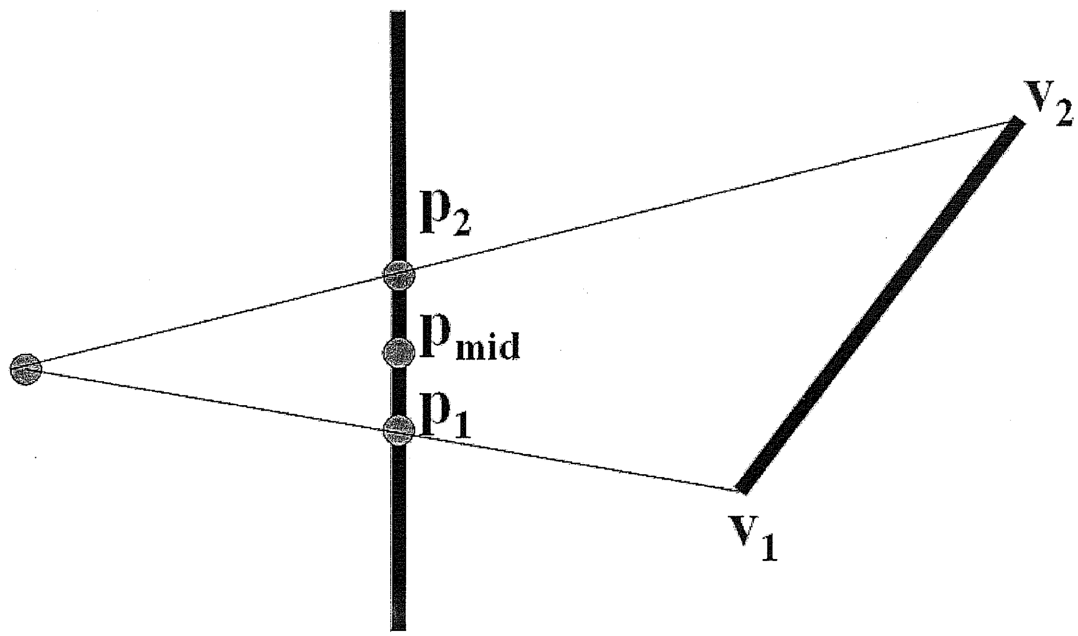
a) **(9 points)** Write out the formula for computing the intensity at vertices v_1, v_2, and v_3.
b) **(8 points)** Derive the intensity of the centroid C ($C = (v_1 + v_2 + v_3)/3$) of the triangle using Gouroud shading.
c) **(8 points)** Derive the intensity of C using Phong shading.

A2. Graphics (25 points) Texture Mapping.

a) **(8 points)** What is summed area table used for? Give the summed area table of the following texture.

2	4	7	5
3	6	1	0
4	1	2	3
2	5	7	9

b) **(7 points)** What are the advantages and disadvantages of using the summed area tables compared with the mipmaps?



c). **(10 points)** A line segment v_1v_2 in 2D space maps to p_1p_2 in a 1D camera, as shown in the figure above. Assume $v1 = (x_1, z_1)$, $v2 = (x_2, z_2)$, p_1 has texture coordinate u_1 and p_2 has texture coordinate u_2. What is the texture coordinate for the midpoint p_{mid} of p_1 and p_2. Assume the camera is at the origin and the image plane is at $z = 1$.

A3. Graphics (25 points) HSR and Transformations

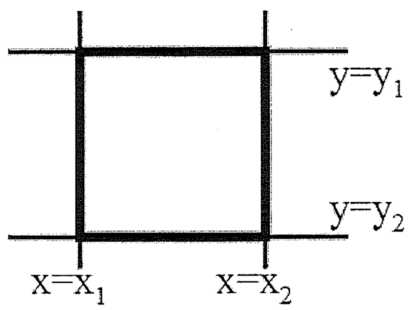
(a) **(5 points)** What's the point of using two color buffers, one to draw into, while the other is being displayed? In OpenGL, how do you enable two buffers?

(b) **(8 points)** Define the term depth buffer (z buffer) and explain how a depth buffer can be used to solve the hidden surface problem? If a scene consists of M polygons, and the image has a resolution of NxN pixels, compare the computational overhead of depth buffer and the ray casting (tracing) algorithm. Be specific on the computational cost of each sub-step.

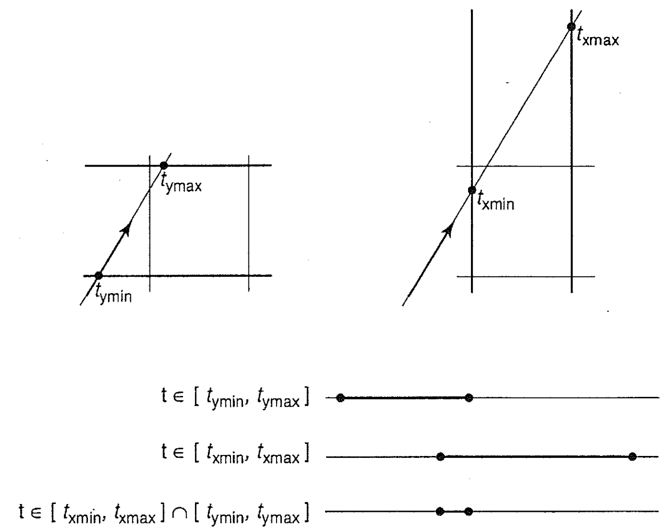
(c) **(7 points)** Compare and contrast BSP and Depth Buffer approaches of hidden surface removal, with special emphasis on their advantages and disadvantages

(d) **(5 points)** Write the transformation **T** mapping $[x, y, z, w]$ to $[x, y, w]$ and prove that **T** is an affine transformation.

A4. Graphics (25 points) Ray-Box Intersection



(4.1)



(4.2)

In this problem, we consider intersecting a ray with a 2D bounding box. A 2D box is defined by two horizontal ($y = y_1$, $y = y_2$) and two vertical lines ($x = x_1$, $x = x_2$) as shown in Figure 4.1. A ray r is defined as $(x_0, y_0) + t\,(x_d, y_d)$.

a) **(7 points)** To determine if r intersects the box, we need to first compute the intersection points of r with the two horizontal lines and the two vertical lines, as shown in Figure 4.2. Derive t_{xmin}, t_{xmax}, t_{ymin}, t_{ymax}.

b) **(7points)** Prove that ray r hits the box if and only if the intervals $[t_{xmin}, t_{xmax}]$ and $[t_{ymin}, t_{ymax}]$ overlap. (This may sound like an algorithm question, but the proof should be very simple and straightforward).

c) **(5points)** What if $x_d = 0$? What if $y_d = 0$? Write out the steps for treating these two cases.

d) **(6points)** Briefly discuss how to extend the 2D algorithm to handle 3D axis aligned bounding boxes (i.e., boxes defined by two x planes, two y planes, and two z planes).