Directional Wireless Sensor Networks with Guaranteed and Coverage and Connectivity

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I. INTRODUCTION

To successfully perform the assigned sensing tasks by a wireless sensor network, sensors must both form a connected network via multi-hop wireless communication, and cover the entire working area or some specific locations. Therefore, deploying a wireless sensor network to provide *connected coverage* becomes a critical research issue. Most of the existing work [1] on the connected coverage problem typically assumes omni-directional sensors with disk-like sensing coverage. However, in real deployment, certain sensors may only possess limited sensing coverage, such as sector (or directional) sensing coverage, due to equipment constraints or environmental impairments. In this work, we investigate how to efficiently deploy directional sensors to form connected networks to cover a set of point-locations or the entire target area. To our best knowledge, this is the first effort to address such problems.

II. NETWORK MODEL AND PROBLEM FORMULATIONS

We consider stationary, directional wireless sensors, whose sensing coverage is a sector centered at each sensor with a sensing radius r_s and a sensing angle α. Moreover, since the directional sensing capability is orthogonal to a sensor's communication capability, each sensor is still assumed to have an omni-directional communication coverage, which is a circle centered at each sensor with a communication radius r_c.

In this work, we assume that the sensing angle α is within interval $[0, \pi]$. Also, for the ease of description, we assume the communication radius is equal to the sensing radius, $i.e.$ $r_s = r_c$, and use r to stand for both, and use $s(r, \alpha)$ to represent the sector sensing coverage. Note that the algorithms and analysis presented in this poster can be easily extend to situations where $r_s \neq r_c$.

We study the following two connected coverage problems for directional wireless sensor networks.

- *Connected Point-Coverage Problem*, which seeks to deploy a minimum number of directional sensors to form a connected network to cover a given set of point-locations scattered in a 2D plane.
- *Connected Region-Coverage Problem*, which seeks to find an efficient pattern to deploy directional sensors to form a connected network to cover an infinite 2D plane.

III. CONNECTED POINT-COVERAGE PROBLEM

In this section, we present an algorithm for the connected point-coverage problem. Since this problem is a generalization of the well-known Geometric Disk Cover problem, this problem is NP-hard, and thus we turn to find an approximation algorithm and derive the algorithm's approximation ratio.

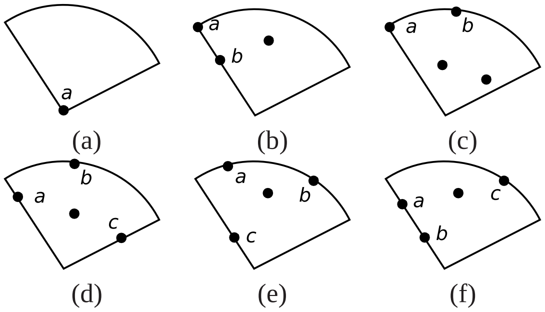
(a) (b) (c)
(d) (e) (f)

Fig. 1. Examples of Sectors Anchored by One, Two and Three Points

In order to tackle the connected point-coverage problem, we first need to solve another NP-hard problem, namely the Geometric Sector Cover problem, which seeks to use a minimum number of sectors $s(r, \alpha)$ to cover a given point set P. We start with following definition.

Definition 3.1: [*Anchored Sector*]: A sector s_i is an anchored sector if it satisfies one of the following three conditions: (1) s_i covers one point a, and a coincides with the center of s_i, as shown in Figure 1(a); (2) s_i covers two or more points, and there exist two points a and b such that a and b are on the same edge (arc) of s_i, and a coincides with one of the end points of this edge (arc), and there exist no other points located on the other edge (arc), as shown in Figures 1(b) and 1(c); (3) s_i covers three or more points, and there exist three points a, b and c such that a, b and c are on different edges (arc); or a and b are on the same edge (arc) of s_i, while c is on one of the other edges (arc), as shown in Figures 1(d), 1(e) and 1(f).

Algorithm 1 presents our algorithm for the Geometric Sector Cover problem, and Theorem 3.2 presents the approximation ratio of Algorithm 1[1].

Theorem 3.2: Let P be a set of points, and $s(r, \alpha)$ be a sector. Algorithm 1 is an $O(log|P|)$-approximation algorithm in terms of the number of sectors $s(r, \alpha)$ required to cover P.

With Algorithm 1, we are now able to present our algorithm for the connected point-coverage problem, which consists of two steps. In the first step, we execute Algorithm 1 on the given set of point-locations. In the second step, we connect all of the sensors deployed in the first step. The complete algorithm is given in Algorithm 2, and Theorem 3.3 gives its approximation ratio.

Theorem 3.3: Let P be a set of points, and let $s(r, \alpha)$ be a directional sensor. Algorithm 2 is an $O(log|P|)$-approximation

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[1] We also developed an approximation algorithm with constant approximation ratio for fat sectors where $sin(\alpha) \geq c$ (c is a positive constant).

Algorithm 1 Greedy Algorithm for Geometric Sector Cover

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INPUT: Sector $s(r, \alpha)$ and a set of points $P$.
OUTPUT: A set of sectors $S$ that covers $P$.
$R \leftarrow P$; $S \leftarrow \phi$; $i = 1$;
While $R$ is not empty
    • Find a sector $s_i(r, \alpha)$ anchored by one, two, or three points in $R$ that covers a maximum number of points in $R$, and let $R_i$ be the set of points of $R$ that are covered by $s_i$;
    • $S = S \cup \{s_i\}$;
    • $R = R \setminus R_i$;
    • $i = i + 1$;
End While
Output $S$;
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Algorithm 2 Algorithm for Connected Point-Coverage Problem

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INPUT: Sensor $s(r, \alpha)$ and a set of locations $P$.
OUTPUT: A set of connected sensors $S$ that covers $P$.
$S \leftarrow \phi$;
Execute Algorithm 1 on $P$, and put the sensors in $S$;
Find a Geometric Minimum Spanning Tree $T$ over $\{p_i\}$ ($p_i$ is the center of $s_i$, $s_i \in S$);
For each edge $p_ip_j$ in $T$, if its length is longer the $r$, then starting from $p_i$ along the edge $p_ip_j$, put one sensor for every distance $r$ until $p_i$ is connected with $p_j$, and put all of the sensors in $S$.
Output $S$;
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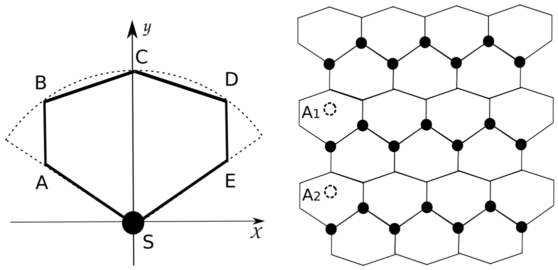
algorithm in terms of the number of sensors $s(r, \alpha)$ to form a connected network to cover P.

IV. CONNECTED REGION-COVERAGE PROBLEM

It is obvious that we need to deploy an infinite number of sensors to cover a 2D plane. Moreover, there must exist some overlaps among the sensing range because the sensing range of each sensor is a sector. Unfortunately, finding the optimal pattern to cover a 2D plane with sectors is still an open problem even if we remove the connectivity constraint. Therefore, we focus on finding an efficient deployment pattern so that the average overlapping area of each sensor is bounded.

We start with some definitions. Given a convex body k, a $packing$ with k is a set $P_k = \{k_i\}$ (k_i is congruent to k) such that all of the convex bodies in P_k are mutually disjoint; a $covering$ with k is a set $C_k = \{k_i\}$ (k_i is congruent to k) such that the union of all of the convex bodies in C_k is the entire 2D plane; a set $T_k = \{k_i\}$ (k_i is congruent to k) is a $tiling$ with k if T_k is both a $packing$ and a $covering$ with k, and k is called a convex tile if a tiling with k exists. A p-hexagon [2] is a hexagon with a pair of parallel opposite sides of equal length. Under this definition, extreme cases are allowed, in which adjacent sides are collinear, and some sides, even the parallel ones, may be reduced to points. Thus, each triangle is a p-hexagon, and so is each quadrilateral and each pentagon that has a pair of parallel sides. It is shown in [3] that each p-hexagon is a *convex tile*.

The best result in the literature so far for covering a 2D plane with sectors is given in [2], as shown in Theorem 4.1. However, the result in [2] does not specifically consider sectors. We extend the method in [2] and find that for sectors, the upper-bound ratio of the area of inscribed p-hexagon over the area of the sector can be improved to approximately 82.7%. Our approach to finding an inscribed p-hexagon is formally described below, and Theorem 4.2 states our result.



(a) (b)

Fig. 2. (a) An inscribed p-hexagon in a sector. (b) A tiling pattern of the 2D plane with the inscribed p-hexagons, where the black dots represent deployed sensors.

• *Find a p-hexagon in a sector*: As shown in Figure 2(a), for sector $s(r, \alpha)$, S and C denote the center of the sector and the middle point of the arc, respectively. We put this sector on an xy-coordinate system where S coincides with the origin, and C is on the y-axis. We find two points A and E on the two edges of the sector with equal distance to S. Then we find two more points B and D on the arc such that AB, DE is vertical to the x-axis. The hexagon *SABCDE* is a p-hexagon.

Given such an inscribed p-hexagon within a sector as shown in Figure 2(a), the corresponding deployment pattern using such p-hexagons is shown in Figure 2(b). Note that a tiling of the 2D plane with inscribed p-hexagons does not always lead to a connected network. Therefore, additional sensors, termed *relay sensors*, such as nodes A_1 and A_2 in Figure 2(b), need to be deployed to guarantee connectivity. Since the number of relay sensors is vanishingly small comparing with the total number of sensors, the overlapping areas introduced by relay sensors can be omitted.

Theorem 4.1: [2] Each convex body k contains a p-hexagon h such that the area of h is no less than $\frac{2\sqrt{3}+3}{8}$ ($\approx 80.8\%$) of the area of k.

Theorem 4.2: Each sector s contains a p-hexagon h such that the area of h is no less than $\frac{3\sqrt{3}}{2\pi}(\approx 82.7\%)$ of the area of s.

V. CONCLUSION AND FUTURE WORK

We present an $O(log|P|)$-approximation algorithm for the connected point-coverage problem. We also give a solution to find an inscribed p-hexagon in sectors along with the tiling pattern of such p-hexagons to form a connected network to cover a 2D plane. Our solution produces less average overlapping area than existing solutions.

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