CIS4/681 – Homework 3 – Logic and Resolution

Due: Thursday May 1, 2008 – 50 points

Solutions

1. (10 points)

   Write a set of inference rules (i.e., implication statements) that would allow the general inheritance of properties down the hierarchy. These rules should involve ISA and HAS predicates. In particular, what inference rules would be needed to infer:

   HAS(tweety, color, yellow)
   HAS(tweety, flies, yes)
   HAS(tweety, eats, seeds)

   Ans.

   1. $(\forall x)(\forall y)(\forall z)((ISA(x,y) \land ISA(y,z)) \Rightarrow ISA(x,z))$
   2. $(\forall x)(\forall y)(\forall z)(\forall w)((ISA(x,y) \land HAS(y,z,w)) \Rightarrow HAS(x,z,w))$

2. (5 points) For each of the following sets of literals, find a most general unifier or state the reason why the set is not unifiable (Note: P and Q are predicate symbols, f and g function symbols, a and b constant symbols, and x, y, and z variable symbols):

   (a) $\{P(x,y), P(a,z)\}$

   Ans.

   $$\textsc{Unify}(P(x,y), P(a,z)) = \{x/a, y/z\}$$

   (b) $\{P(x,x), P(a,b)\}$

   Ans.

   $$\textsc{Unify}(P(x,x), P(a,b)) = \mathit{fail}$$

The unification fails because x cannot take on the value a and b at the same time.

(c) $\{P(x, y), P(a, f(a))\}$

Ans.

$$\text{UNIFY}(P(x, y), P(a, f(a))) = \{\text{x/a, y/f(a)}\}$$

(d) $\{P(x, f(x)), P(a, b)\}$

Ans.

$$\text{UNIFY}(P(x, f(x)), P(a, b)) = \textit{fail}$$

The unification fails because, if x/a, then $f(a)$ conflicts with the value b.

(e) $\{P(x, f(x), a), P(b, y, x)\}$

Ans.

$$\text{UNIFY}(P(x, f(x), a), P(b, y, x)) = \textit{fail}$$

The unification fails because x cannot take on the value a and b at the same time.

3. (10 points) Put the following predicate-calculus formulas into clausal form:

(a) $(\forall x)(\forall y)((P(x) \land Q(y)) \Rightarrow (\exists z)(R(x, y, z)))$

Ans.

0. $(\forall x)(\forall y)((P(x) \land Q(y)) \Rightarrow (\exists z)(R(x, y, z)))$
1. $(\forall x)(\forall y)(\neg(P(x) \land Q(y)) \lor (\exists z)(R(x, y, z)))$ ... Eliminate implications
2. $(\forall x)(\forall y)((\neg P(x) \lor \neg Q(y)) \lor (\exists z)(R(x, y, z)))$ ... Move $\neg$ inwards

3. $(\forall x)(\forall y)(\exists z)((\neg P(x) \lor \neg Q(y)) \lor R(x, y, z))$ ... Move quantifiers left
4. $(\neg P(x) \lor \neg Q(y)) \lor R(x, y, F(x, y))$ ... Skolemize
5. $\underline{\neg P(x) \lor \neg Q(y) \lor R(x, y, F(x, y))}$ ... Flatten Nested conjunctions

(b) $(\exists x)(\forall y)(\exists z)(P(x) \Rightarrow (Q(y) \Rightarrow R(z)))$

Ans.

0. $(\exists x)(\forall y)(\exists z)(P(x) \Rightarrow (Q(y) \Rightarrow R(z)))$
1. $(\exists x)(\forall y)(\exists z)(\neg P(x) \lor (\neg Q(y) \lor R(z)))$ ... Eliminate implications
2. $(\forall y)(\exists z)(\neg P(C) \lor (\neg Q(y) \lor R(z)))$ ... Skolemize
3. $\neg P(C) \lor (\neg Q(y) \lor R(F(y)))$ ... Skolemize
4. $\underline{\neg P(C) \lor \neg Q(y) \lor R(F(y))}$ ... Flatten Nested conjunctions

(c) $(\forall x)(P(x) \Rightarrow (\exists y)(Q(x, y))) \land (\forall x)(\neg P(x) \Rightarrow \neg(\exists y)(Q(x, y)))$

Ans.

0. $(\forall x)(P(x) \Rightarrow (\exists y)(Q(x, y))) \land (\forall x)(\neg P(x) \Rightarrow \neg(\exists y)(Q(x, y)))$
1. $(\forall x)(\neg P(x) \lor (\exists y)(Q(x, y))) \land (\forall x)(P(x) \lor \neg(\exists y)(Q(x, y)))$ ... Eliminate implications
2. $(\forall x)(\neg P(x) \lor (\exists y)(Q(x, y))) \land (\forall x)(P(x) \lor (\forall y)(\neg Q(x, y)))$ ... Move $\neg$ inwards
3. $(\forall x_1)(\neg P(x_1) \lor (\exists y_1)(Q(x_1, y_1))) \land (\forall x_2)(P(x_2) \lor (\forall y_2)(\neg Q(x_2, y_2)))$
   ... Standardize variables
4. $(\forall x_1)(\exists y_1)(\forall x_2)(\forall y_2)((\neg P(x_1) \lor Q(x_1, y_1)) \land (P(x_2) \lor \neg Q(x_2, y_2)))$
   ... Move quantifiers left
6. $(\forall x_2)(\forall y_2)((\neg P(x_1) \lor Q(x_1, F(x_1))) \land (P(x_2) \lor \neg Q(x_2, y_2)))$ ... Skolemize
7. $\underline{(\neg P(x_1) \lor Q(x_1, F_1(x_1))) \land (P(x_2) \lor \neg Q(x_2, y_2))}$ ... Skolemize

4. (25 points) In this part I want you to use resolution in order to implement Sir Bedevere's proof that the girl the villagers were trying to burn was, indeed, a witch.

   - Every woman that burns is a witch.
   - The girl is a woman.

- Anything that is made of wood burns.
- Anything that floats is made of wood.
- The duck floats.
- Given any two things, if one of them float and if the same weight as the other, then the other thing floats.
- The girl and the duck are the same weight.

(a) Translate these sentences into formulas in predicate logic.

Ans.

- $(\forall x)((\text{Woman}(x) \wedge \text{Burns}(x)) \Rightarrow Witch(x))$
- $\text{Woman}(g)$
- $(\forall x)(\text{Made-of-Wood}(x) \Rightarrow \text{Burns}(x))$
- $(\forall x)(\text{Float}(x) \Rightarrow \text{Made-of-Wood}(x))$
- $\text{Float}(d)$
- $(\forall x)(\forall y)((\text{Float}(x) \wedge \text{Same-Weight}(x, y)) \Rightarrow \text{Float}(y))$
- Same-Weight(g,d)

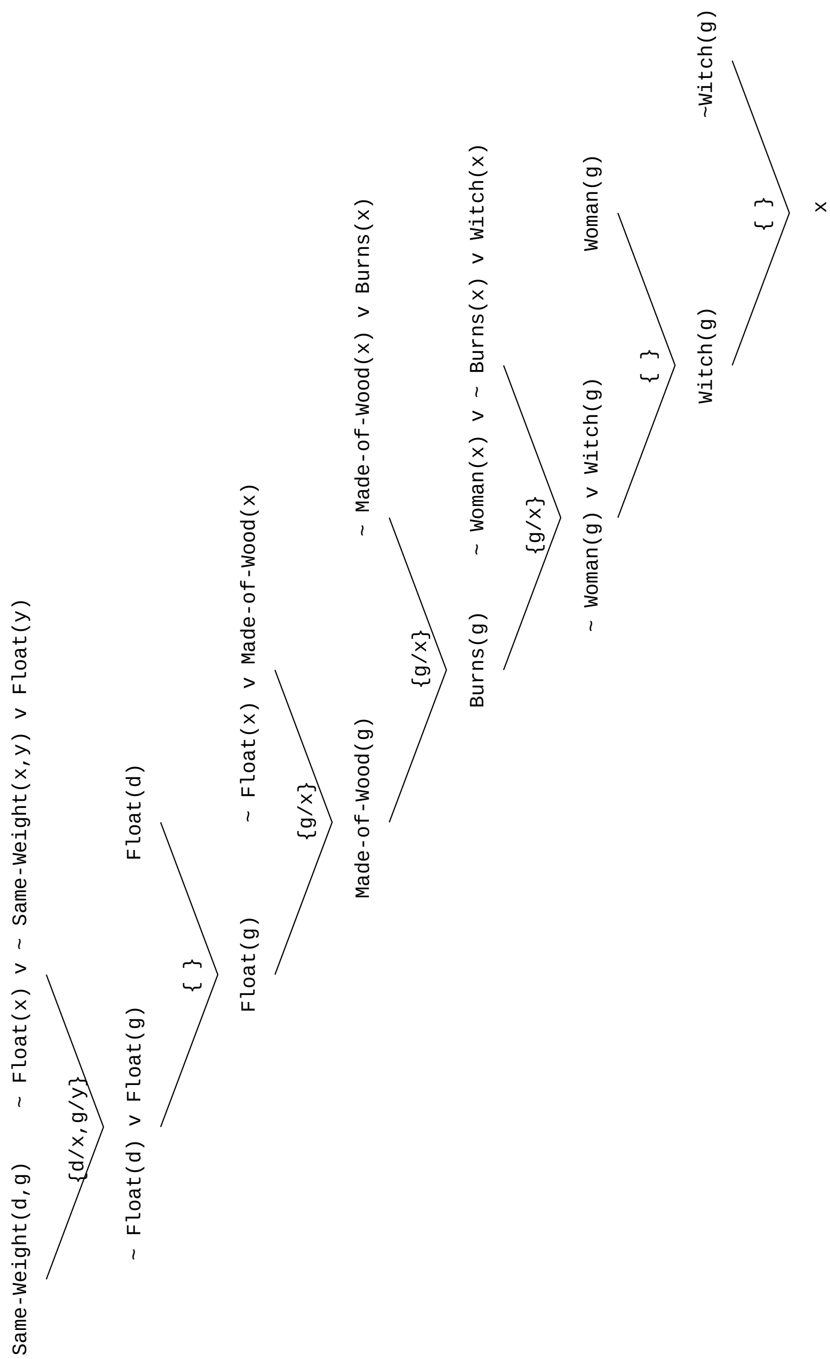
(b) Convert the formulas into clausal form.

Ans.

- $\neg\text{Woman}(x) \vee \neg\text{Burns}(x) \vee Witch(x)$
- $\text{Woman}(g)$
- $\neg\text{Made-of-Wood}(x) \vee \text{Burns}(x)$
- $\neg\text{Float}(x) \vee \text{Made-of-Wood}(x)$
- $\text{Float}(d)$
- $\neg\text{Float}(x) \vee \neg\text{Same-Weight}(x, y) \vee \text{Float}(y)$
- Same-Weight(d,g)

(c) Prove that the girl is a witch using resolution.

Ans.
Suppose $\neg\text{Witch}(g)$, and prove $\text{Witch}(g)$ by contradiction.


Same-Weight(d,g)
~ Float(x) v ~ Same-Weight(x,y) v Float(y)
{d/x,g/y}
~ Float(d) v Float(g)
Float(d)
{ }
Float(g)
~ Float(x) v Made-of-Wood(x)
{g/x}
Made-of-Wood(g)
~ Made-of-Wood(x) v Burns(x)
{g/x}
Burns(g)
~ Woman(x) v ~ Burns(x) v Witch(x)
{g/x}
~ Woman(g) v Witch(g)
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