Logical Agents

Chapter 7

(based on slides from Stuart Russell and Hwee Tou Ng)

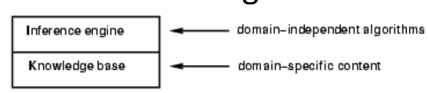
Logical Agents

- Knowledge-based agents – agents that have an explicit representation of knowledge that can be reasoned with.
- These agents can manipulate this knowledge to infer new things at the "knowledge level"

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

Knowledge bases



- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - Tell it what it needs to know
- Then it can Ask itself what to do - answers should follow from the KB
- Agents can be viewed at the knowledge level - i.e., what they know, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

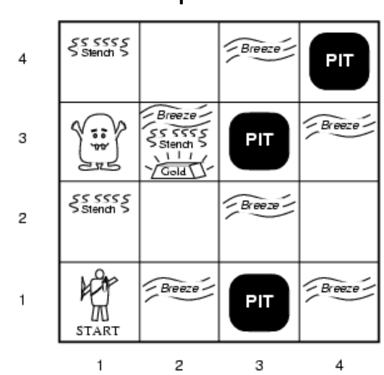
A simple knowledge-based agent

```
function KB-AGENT(percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE(action, t))
   t ← t + 1
   return action
```

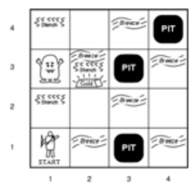
- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

A Wumpus World



Wumpus World PEAS description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment: 4 x 4 grid of rooms
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



- Sensors: Stench, Breeze, Glitter, Bump, Scream (shot Wumpus)
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Wumpus world characterization

- Fully Observable
- Deterministic
- Episodic
- Static
- Discrete
- Single-agent?

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic
- Episodic
- Static
- Discrete
- Single-agent?

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic
- Static
- Discrete
- Single-agent?

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static
- Discrete
- Single-agent?

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete
- Single-agent?

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent?

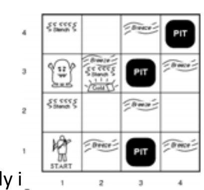
Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

Wumpus World

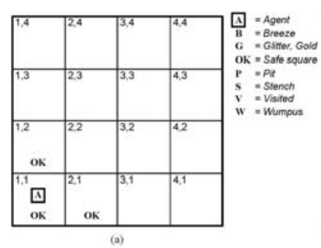
- Percepts given to the agent

1. Stench
2. Breeze
3. Glitter
4. Bumb (ran into a wall)
5. Scream (wumpus has been hit by arrow)



- Principle Difficulty: agent is initially i configuration of the environment – going to have to reason to figure out where the gold is without getting killed!

Exploring the Wumpus World

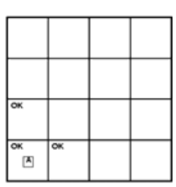


Initial situation:

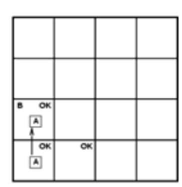
Agent in 1,1 and percept is [None, None, None, None, None]

From this the agent can infer the neighboring squares are safe (otherwise there would be a breeze or a stench)

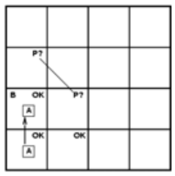
Exploring a wumpus world



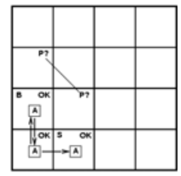
Exploring a wumpus world



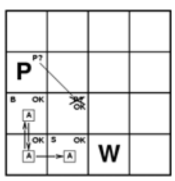
Exploring a wumpus world



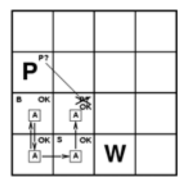
Exploring a wumpus world



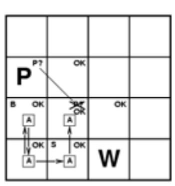
Exploring a wumpus world



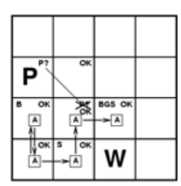
Exploring a wumpus world



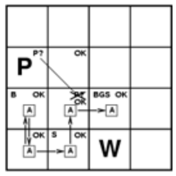
Exploring a wumpus world



Exploring a wumpus world



Exploring a wumpus world



In each case where the agent draws a conclusion from the available Information, that conclusion is guaranteed to be correct if the available Information is correct…

This is a fundamental property of logical reasoning

Logic in general

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines how symbos can be put together to form the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world (given an interpretation)
- E.g., the language of arithmetic
  - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence
  - $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
  - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$

Entailment

- Entailment means that one thing follows logically from another:

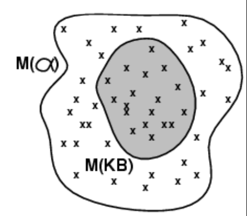
  $KB \models \alpha$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing "the Phillies won" and "the Reds won" entails "Either the Phillies won or the Reds won"
  - E.g., $x+y = 4$ entails $4 = x+y$
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$

  E.g. *KB* = Phillies won and Yankees won α = Phillies won

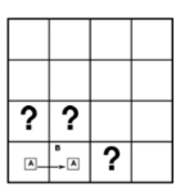


Entailment in the wumpus world

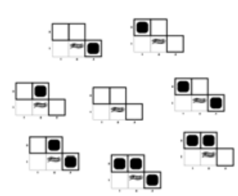
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Consider possible models for *KB* assuming only pits

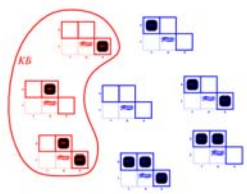
3 Boolean choices ⇒ 8 possible models



Wumpus possible models

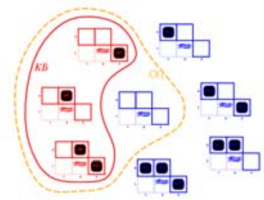


Wumpus models



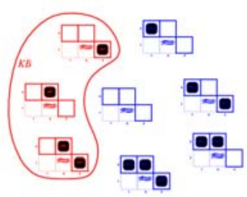
- *KB* = wumpus-world rules + observations

Wumpus models



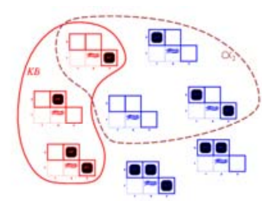
- *KB* = wumpus-world rules + observations
- α_1 = "there is no pit in [1,2]", $KB \models \alpha_1$, proved by model checking

Wumpus models



- *KB* = wumpus-world rules + observations

Wumpus models



- *KB* = wumpus-world rules + observations
- α_2 = "there is no pit in [2,2]", $KB \nvDash \alpha_2$

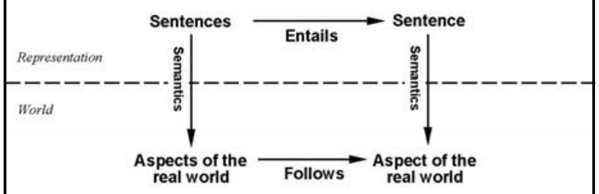
Inference and Entailment

- Inference is a procedure that allows new sentences to be derived from a knowledge base.
- Understanding inference and entailment: think of
  - Set of all consequences of a KB as a haystack
  - α as the needle
- Entailment is like the needle being in the haystack
- Inference is like finding it

Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from *KB* by inference procedure *I*
- Soundness: *i* is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
- Completeness: *i* is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the *KB*.

Step Back…



This is an inference procedure whose conclusions are guaranteed to be true In any world where the premises are true.

If KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world.

Propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas
- The proposition symbols P_1, P_2 etc are (atomic) sentences
  - If S is a sentence, $\neg(S)$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $(S_1 \wedge S_2)$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $(S_1 \vee S_2)$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $(S_1 \Rightarrow S_2)$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $(S_1 \Leftrightarrow S_2)$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ false, $P_{2,2}$ true, $P_{3,1}$ false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$ is true iff S is false
$S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true
$S_1 \vee S_2$ is true iff S_1 is true or S_2 is true
$S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true
i.e., is false iff S_1 is true and S_2 is false
$S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (true \vee false) = true \wedge true = true$$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

John likes football and John likes baseball.

John likes football or John likes baseball.

(English or is a bit different…)

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

John likes football and John likes baseball.

John likes football or John likes baseball.

If John likes football then John likes baseball.

(Note different from English – if John likes football maps to false, then the sentence is true.)

(Implication seems to be if antecedent is true then I claim the consequence is, otherwise I make no claim.)

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j].
Let $B_{i,j}$ be true if there is a breeze in [i, j].

$\neg P_{1,1}$
$\neg B_{1,1}$
$B_{2,1}$

"Pits cause breezes in adjacent squares"

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

Simple Inference Procedure

- $KB \models \alpha$?
- Model checking – enumerate the models, and check if α is true in every model in which KB is true. Size of truth table depends on # of atomic symbols.
- Remember – a model is a mapping of all atomic symbols to true or false – use semantics of connectives to come to an interpretation for them.

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	false	false	false	false	false	false	false	true
false	false	false	false	false	false	true	false	true
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	false	true
false	true	false	false	false	false	true	<u>true</u>	<u>true</u>
false	true	false	false	false	true	false	<u>true</u>	<u>true</u>
false	true	false	false	false	true	true	<u>true</u>	<u>true</u>
false	true	false	false	true	false	false	false	true
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	false

Inference by enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [])

function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
      if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
      else return true
   else do
      P ← FIRST(symbols); rest ← REST(symbols)
      return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
             TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- For *n* symbols, time complexity is $O(2^n)$, space complexity is $O(n)$

Logical equivalence

- Two sentences are logically equivalent iff true in same models: α ≡ ß iff $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of $\vee$
$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$ implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ de Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ de Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of $\vee$ over $\wedge$

Validity and satisfiability

A sentence is valid if it is true in all models,
e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:
$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model
e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models
e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:
$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

Proof methods

- Proof methods divide into (roughly) two kinds:
  - Application of inference rules
    - Legitimate (sound) generation of new sentences from old
    - Proof = a sequence of inference rule applications
      Can use inference rules as operators in a standard search algorithm
    - Typically require transformation of sentences into a normal form
  - Model checking
    - truth table enumeration (always exponential in *n*)
    - improved backtracking, e.g., Davis--Putnam-Logemann-Loveland (DPLL)
    - heuristic search in model space (sound but incomplete)
      e.g., min-conflicts-like hill-climbing algorithms

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \lor \beta$.
   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:
   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \lor \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law ($\land$ over $\lor$) and flatten:
   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Resolution example

- *KB* = $(B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}$ $\alpha = \neg P_{1,2}$
-