CISC4-681 Useful Rules for Probability

1. Conditional Probabilities

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

2. Product Rule

$$P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$$

3. Chain Rule (multiple applications of product rule)

$$P(x1, \ldots, xn) = P(xn|xn-1, \ldots, x1)P(xn-1, \ldots, x1)$$
$$= P(nx|xn-1, \ldots, x1)P(xn-1|xn-2, \ldots, x1) \ldots P(x2|x1)P(x1)$$
$$= \prod_{i=1}^{n} P(xi|xi-1, \ldots, x1)$$

4. Two random variables with values a and b are **independent** (also marginal independence and absolute independence):

   $P(a \wedge b) = P(a)P(b) \quad or \quad P(a|b) = P(a) \; or \; P(b|a) = P(b)$

   Independence between variables A and B can be written as:

   $\boldsymbol{P}(X,Y) = \boldsymbol{P}(X)\boldsymbol{P}(Y) \; or \; \boldsymbol{P}(X|Y) = \boldsymbol{P}(X) \; or \; \boldsymbol{P}(Y|X) = \boldsymbol{P}(Y)$

5. Two random variables A and B are **independent given C** if

   $\boldsymbol{P}(A \wedge B|C) = \boldsymbol{P}(A|C)\boldsymbol{P}(B|C) \; or \; \boldsymbol{P}(A|B,C) = \boldsymbol{P}(A|C)$

6. Marginalization or summing out. Suppose a query involves a single variable X. Let **E** be the list of evidence variables, let **e** be the list of observed values for them, and let **Y** be the remaining unobserved variables (hidden variables).

   $\boldsymbol{P}(X|e) = \alpha\boldsymbol{P}(X,\boldsymbol{e}) = \alpha \sum_{y} \boldsymbol{P}(X,\boldsymbol{e},y)$

   Where the summation is over all possible combinations of values of the unobserved variables **Y**.

7. Bayes Rule

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)}$$

Generally this is useful when we have good probability estimates for 3 numbers, and want to compute the 4^{th}. For example, if we perceive as evidence the effect of some unknown cause we would like to determine the cause. So,

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

According to the text, here $P(effect|cause)$ quantifies the relationship in the causal direction where $P(cause|effect)$ describes the diagnostic direction. E.g., in medical diagnosis the doctor often knows conditional probabilities on causal relationships such as $P(effect|cause)$, and wants to derive the diagnosis.